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CHAOTIC DYNAMICS OF THE SOLAR CYCLE

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16. Abstract (Limit: 200 words)

The cyclic variation of solar activity is both irregular and intermittent. We have sought to isolate and illuminate the physical mechanisms of this behavior and to provide a mathematical description of it. Our work has brought out three ingredients of the solar cycle that we believe to be central to its operation. (1) The seat of the solar cycle is in a shear layer just below the solar convection zone. We have investigated the structure of this layer (which we call the solar tachocline) in some detail. (2) The spatio-temporal development of the solar cycle is represented by the propagation of robust solitary waves which are affected by dissipation and instability. We have studied the structure and interactions of such waves, which we call solitoids. (3) On top of the simple propagative behavior of the solar solitoids there are intermissions during which the number of sunspots remains quite small. We attribute these intermissions (such as the Maunder minimum) to a form of interaction between the convection zone and the tachocline which is characteristic of a process that we have developed and that we call on/off intermittency. These three ingredients make up some of the key features of the solar cycle and may be expected to play a role in future simulations of the solar cycle.

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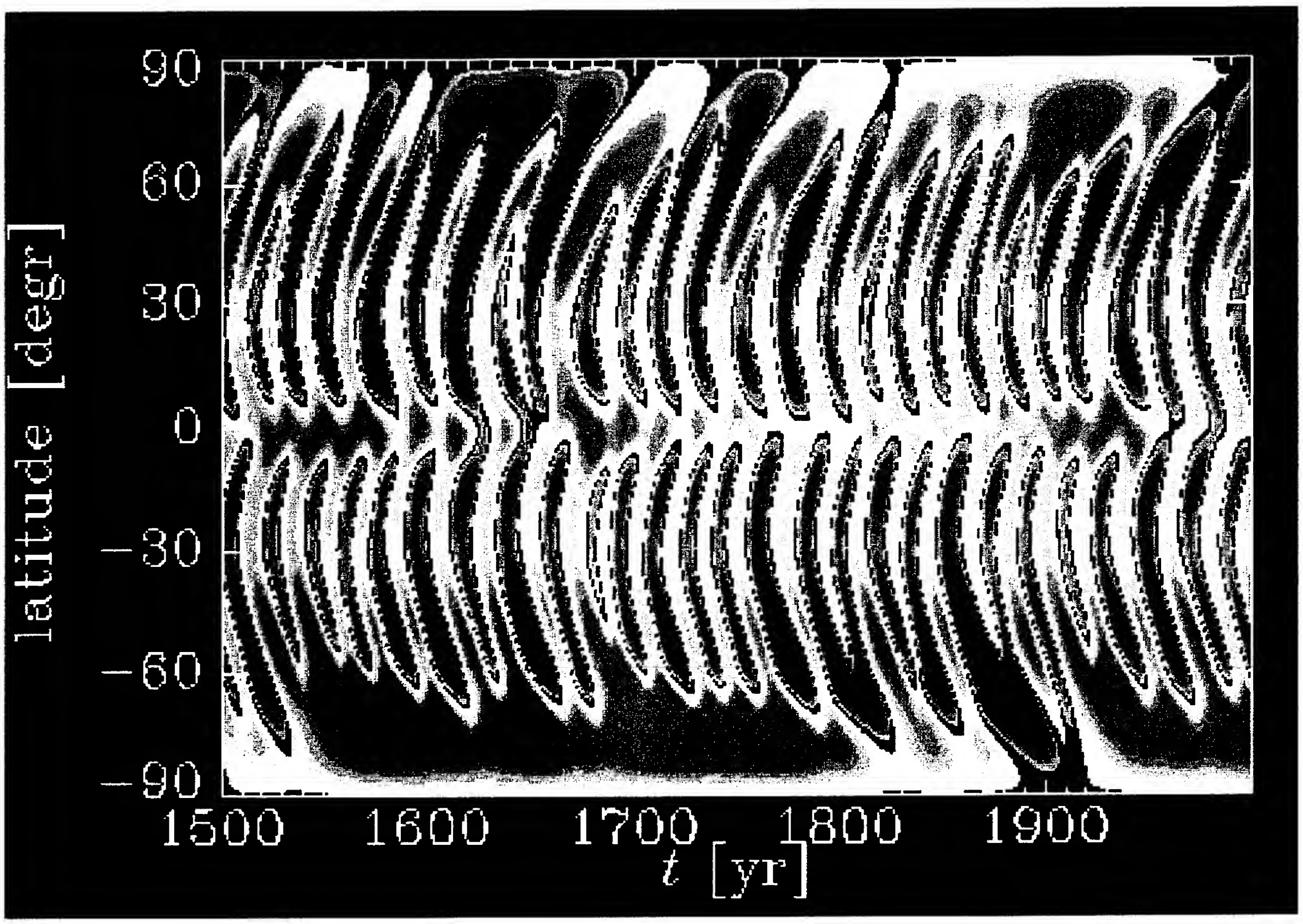

A Theoretical Butterfly Diagram (from "Modeling a Maunder Minimum," by A. Brandenburg and E.A. Spiegel, in preparation).

Final Technical Report

CHAOTIC DYNAMICS OF THE SOLAR CYCLE

February 1996

The Goal

The existence of sunspots has been known for millenia. It has been known for sixty years that spots on the sun occur when strong magnetic flux tubes protrude from the solar surface. The strong fields locally inhibit convection and so reduce the energy flow from the sun. The region of reduced heat flux naturally is darker than its surroundings and is called a sunspot.

In the middle of the last century it was discovered that there is a cyclic variation of the sunspot number. This variation consists of a rise and fall of the number of spots on the sun in an 11.1-year cycle which is not strictly periodic. Moreover, the location of the principal solar activity, as demarked by sunspots, is localized in space and time to a band of latitudes that drifts from midlatitude to the equator over the course of the solar cycle. This spatio-temporal variation is expressed in the so-called butterfly diagram.

The aim of our project has been to isolate the essential physical causes of the cyclic behavior of spot formation and to pursue their mathematical and physical ramifications for the sun, for other stars and in the more general context of nonlinear science. The main results obtained in this effort are these: (1) A localization of the seat of solar activity to a layer just beneath the solar convection zone. We call this layer the solar tachocline. (2) The explanation of the butterfly diagram of solar activity in space and time in terms of nonlinear solitary waves (which we call solitoids) and the development of the theory of such waves in several directions. (3) An explanation of the dearth of sunspots for seventy-five years in the time of Louis XIV by a mathematical mechanism we have called on/off intermittency and the connection of this process to the solar tachocline by a simple dynamical model.

In this report, we briefly outline these three developments. We also add a bibliography below that refers to the several other projects that grew out of these studies. A more extensive summary is in preparation for presentation in the Enrico Fermi School of Physics this June and will be forwarded at that time.

The Solar Tachocline

The fact that solar activity is, at any time, restricted to a fairly narrow band of latitude has led us to develop the idea that the source of sunspots is confined to a layer in the sun of comparable, if somewhat smaller, depth than the width of the excitability band. The convection zone of the sun occupies the outer third of the solar radius and is therefore too deep to meet this requirement. We have therefore developed the idea that sunspots form in a layer just below the convection zone. The existence of such a layer, adumbrated twenty-five years ago, has now been confirmed by helioseismology and we have called it the solar tachocline. It mediates the transition from the outer, strong differential rotation to the more rigid rotational behavior of the inner sun.

In our picture, the active solar convective layer, which occupies the outer third of the solar radius, is the analogue of the earth's atmosphere while the inner, deep core of the sun corresponds to the earth's oceans. In this analogy, the tachocline corresponds to the oceanic thermocline, and this is the source of its name. We have made a preliminary theory of its structure which is in fair agreement with the best results of helioseismology. The solar cycle is then a sort of magnetic weather system of the sun in which magnetic rainfall descends from the convection zone only to be returned from the tachocline in the form of magnetic storms — the sunspots.

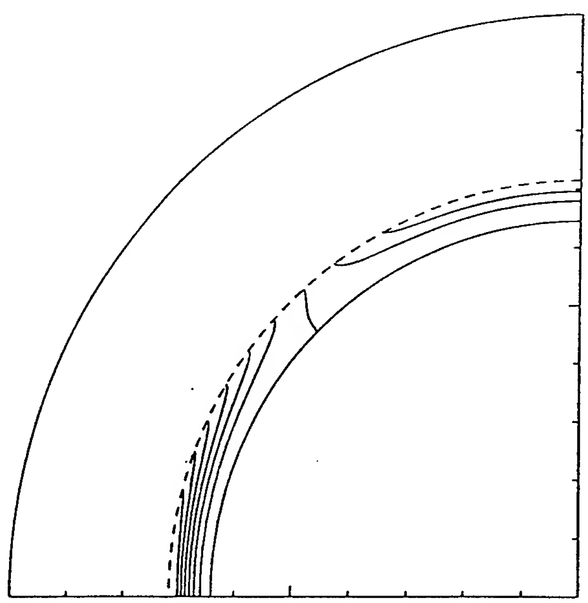

Fig. 1. Structure of the Solar Tachocline (from "The Solar Tachocline" by E.A. Spiegel and J.P. Zahn, *Astron. & Astrophys.*, **265**, 106-114, 1993).

It now seems to be generally accepted that the failure of early simulations directed at reproducing the solar cycle accurately may be due to the omission of a tachocline from the modeling. In the wake of recent highly successful simulations of the geodynamo by Glatzmaier and Roberts we may hope that the solar problem may soon be similarly studied and that the inclusion of a tachocline will be a key ingredient just as inclusion of a solid core was essential in the terrestrial work.

Solar Solitoids

The plot of the latitude of solar activity as a function of time that is called the butterfly diagram shows that solar activity occurs in a limited range of latitude that drifts during the solar cycle from midlatitude to the equator. There is similar activity in each hemisphere, though in detail there is significant asymmetry about the equator.

Our explanation of the butterfly pattern is that there is a pair of solitary waves, one in each hemisphere, each propagating toward the equator. These nonlinear waves arise because of an a magnetohydrodynamic instability of the tachocline that develops as plunging plumes in the convection zone bring magnetic field down to be intensified by the shear of the tachocline. It is in the nature of mhd instabilities that they often occur as overstability or growing oscillations. Whatever the details of the mechanism, its nonlinear equation is generically described by a nonlinear equation known as the complex Ginzburg-Landau equation. We have adapted this equation to the case of the tachocline by allowing for latitude dependence of its governing parameters. In the examples of this form of equation that we have studied, solitary waves of the right kind are produced and propagate in a convincing manner toward the equator.

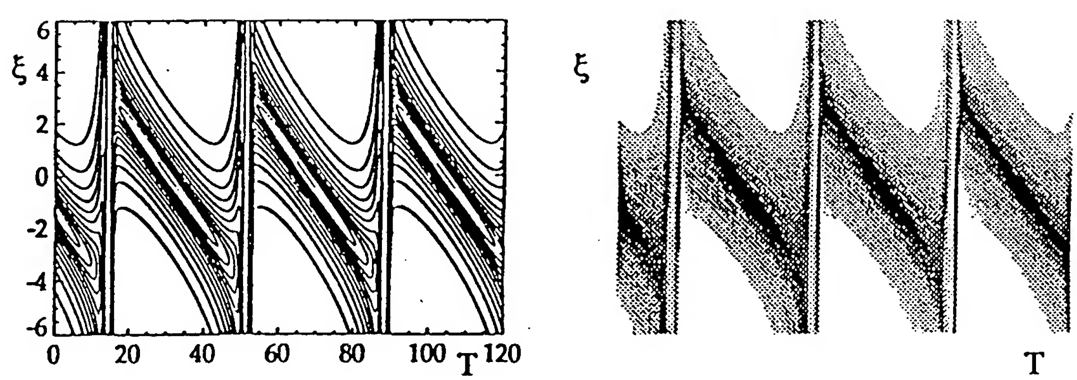

Fig. 2. Propagation of a Solar Solitoid in the Northern Hemisphere (from "Waves of Solar Activity,' by M.R.E. Proctor and E.A. Spiegel, in *The Sun and Cool Stars*, (Springer-Verlag) 1991).

Like solitons that are found in nondissipative integrable systems these solitary waves possess a good deal of stability. Unlike solitons they are subject to dissipation and instability. These extra features make them more interesting in that their parameters (amplitude, breadth, velocity) are not constants, as for solitons, but are subject to variations that may be complicated. In the cases where the dissipation and instabilities are in a reasonable balance on average, the parameters of these solitary waves are governed by dynamical systems which control the behavior of the solitary waves. We call these such solitary waves solitoids.

We have derived equations of motion (dynamical systems) for the solar solitoids that propagate in the tachocline. When we allow for the latitudinal variations of the properties of the tachocline, we find a reasonable description of the butterfly diagram. The situation is enriched when the solitoids in the two solar hemisphere are allowed to interact. In this case, we find mildly irregular behavior and north-south asymmetry as one sees in the observed solar cycle.

To clarify these aspects of the problem we have also made detailed studies of the interactions of solitary waves in general and of the spatio-temporal chaos this produces. The latter theory has a variety of applications in applied mathematics and we have developed the theory of interacting solitoids for such studies, with applications to biomathematics and other branches of astrophysics.

On/Off Intermittency

For nearly three hundred years, the number of spots on the sun at any time has fluctuated up and down on the average time of 11.1 years. This variation resembles that of a chaotic oscillator such as those third-order systems that have been much studied in recent decades. But, during the time of Newton, sunspots became quite rare and this lowering of activity has puzzled investigators ever since. It is especially intriguing because there are hints that this dearth of sunspots, known as the Maunder minimum, is correlated with little ice ages.

We have devised a mechanism to explain the near disappearance of sunspots, one that fits in with our view that the solar cycle originates in the tachocline. The mechanism, called on/off intermittency, is a general one that explains how systems alternate between high activity of a certain kind and quiet periods of inactivity. The alternation of a turbulent fluid between turbulent and laminar phases is an example of this kind of behavior.

A system that exhibits on/off intermittency has two components, an observable *oscillator* that is activated by a *driver* that is a larger system whose variables are neither predictable nor observable. The sudden outbursts of activity of the oscillator cannot be predicted either. In the application of this mechanism to the solar cycle the tachocline is the observable oscillator and the solar convection zone is the driver. The magnetic field that rains down from the convection zone enters the tachocline

where it is twisted into a strong toroidal field by the characteristically strong shear across that layer. Since the density is stably stratified in the tachocline, the field is confined there until it becomes strong enough to erupt into the convection zone where it is buoyed up to form sunspots on the surface. If the level of magnetic rainfall drops below a certain critical level, the instability of the tachocline does not occur and the main solar cycle ceases, though dynamo activity in the solar convection zone continues. This picture gives rise to the process of on/off intermittency in the sun and is responsible for intermissions of the cycle.

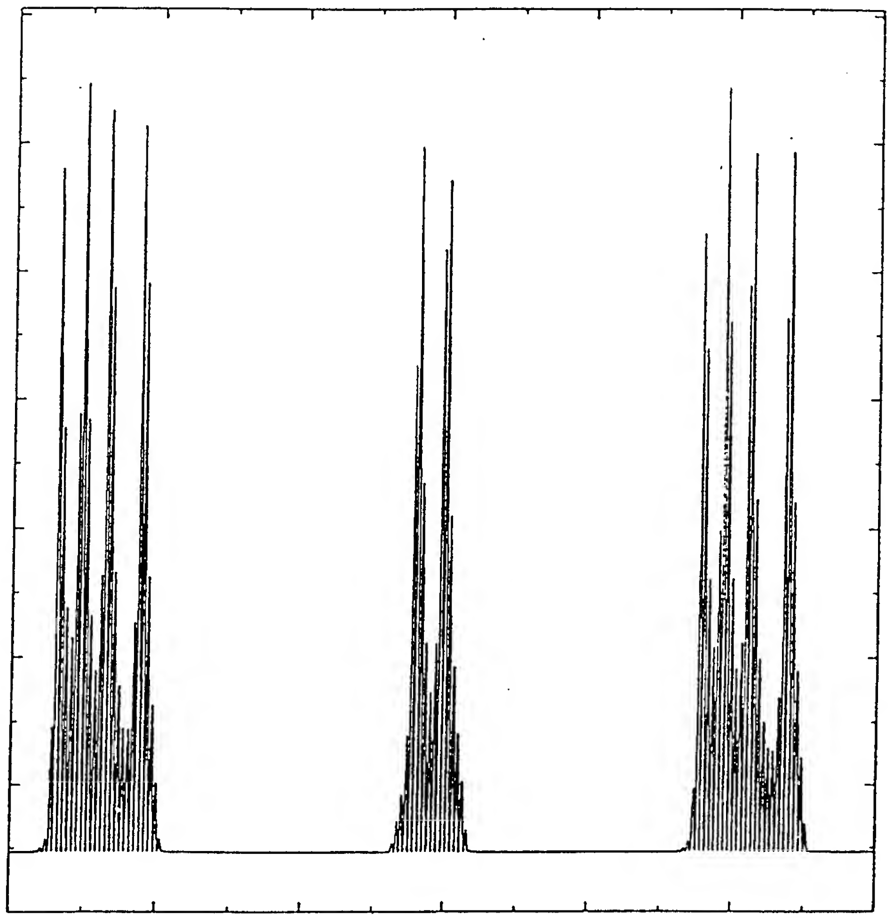

Fig. 3. Output of a Model of Solar Activity Exhibiting On/Off Intermittency, (from "The Intermittent Solar Cycle," by N. Platt, E.A. Spiegel and C. Tresser, *Astrophys. & Geophys. Fluid Mech.*, **73**, 141–161, 1993).

Publications

Each of these ingredients of the solar cycle has been the object of separate lines of investigation and it will need a few more years to work out the details of their intermeshing. We give now a list of papers in which these ideas and related developments appear:

"Waves of solar Activity," (M.R.E. Proctor and E.A. Spiegel), in *the sun and the Cool Stars*, D. Moss, G. Rüddinger and I. Tuominen, eds. (Springer-Verlag) 117-128, 1991.

"Interacting Localized Structures with Galilean Invariance," (C. Elphick, O. Regev, G.R. Ierley and E.A. Spiegel), *Phys. Rev. A*, **44**, 1110-1122, 1991.

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"Convection-induced shears of general planforms," (J. Massaguer, E.A. Spiegel. and J.-P. Zahn) *Phys. Fluids*, **A 4 (7)**, 1333, 1992.

"Circulation and turbulence in rotating stars," (J.-P. Zahn), *Astron. Astrophys.*, **265**, 115, 1992.

"On-off intermittency: A mechanism for bursting" (N. Platt, E.A. Spiegel and C. Tresser) *Phys. Rev. Lett.*, **70**, 279-282, 1993.

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